

# Understanding Inertia Ratio in Synchronous Motor Control

## Executive summary

When sizing a motor for a particular application, the general rule of thumb is to keep the inertia ratio below 10:1—meaning the motor moves a load 10X heavier than its rotor. If the ratio is too high, one solution is to add a gearbox to reduce inertia. However, this increases the amount of energy used. This paper presents a detailed quantitative comparison of response times between low-inertia and medium-inertia motors in both rigid and elastic systems, with calculations of limit criteria for inertia ratios. How to measure energy consumption of gearboxes is explained, showing why choosing the right gearbox can achieve significant savings.

# Introduction

The electromagnetic interaction is the second of the four elementary interactions; it is  $10^{36}$  more intense than the gravitational interaction and is responsible for most visible phenomena: light, electricity, magnetism, etc. Three-phase synchronous motors work using this interaction: the wire-wound stator creates a rotating magnetic field which drives the permanent magnets fixed to the rotor. Indeed the magnetic field of the rotor is constantly trying to align with the stator field to minimize the system energy.

In order to size the motor for an application, the inertia ratio is usually kept below 10:1. This means that the motor moves a load which is ten times heavier than its rotor. It is a kind of “rule of thumb” to obtain a response of a system which is acceptable. When this ratio is too high, it is possible to add a gearbox that reduces the apparent inertia of the load to the views of the motor.

With the controller integrated into the servo-drive Lexium 32, Schneider Electric is increasing the maximum allowable inertia ratio. This reduces the size of motors, and sometimes allows doing without gearbox.

# Inertia issue

## Inertia

The inertia of an object is its ability to maintain a constant speed. More the inertia of an object is large, the greater must be the force required to move it.

## Inertia ratio

The inertia ratio is the ratio between the inertia of the load and the one of the motor. The inertia of the motor includes the rotor and the shaft. It depends on the shape and the weight of materials used.

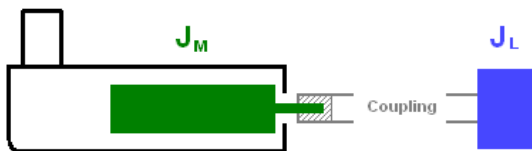


Figure 1: Inertias in movement

$$\text{Inertia Ratio} = \frac{J_L}{J_M}$$

Where:

- $J_L$  The inertia of the load in [kg.m<sup>2</sup>]
- $J_M$  The inertia of the motor in [kg.m<sup>2</sup>]

It is often expressed as X:1, where X is the result of the division. This ratio provides an insight of the importance of the load moved by the motor.

Manufacturers of electric motors and servo-drives suggest keeping this ratio below 20:1, or 10:1. Indeed beyond these values, the rotor is carried away by the load, whereas this is the contrary which is desired.

## Gearbox

The addition of a gearbox between the load and the motor allows reducing the apparent inertia, to the views of the motor.

$$J_{L, \text{ motor side}} = \frac{J_L}{R^2}$$

Where:

- $J_{L, \text{ motor side}}$  The reflection of the load inertia, on the motor side in [kg.m<sup>2</sup>]
- $J_L$  The load inertia in [kg.m<sup>2</sup>]
- $R$  The reduction ratio of the gearbox, without unit [ ]

Thus, with a load of 100 [kg.m<sup>2</sup>], and a motor of 1 [kg.m<sup>2</sup>] the inertia ratio would be of 100:1. With the addition of a gearbox with a ratio of 5:1, this reduces the inertia ratio to 4:1. It is therefore easier to move the load with a gearbox while keeping a reasonable size of the motor. However, that will have significant repercussions on the electrical consumption.

## Rigid systems

When the link between the motor and the load is rigid, the two inertias are not separable. This should be seen as one piece, where the inertia is the sum of the inertias of the motor and of the load.



Figure 2: Rigid system

The performance only depends on the motor torque. If the latter is important, the system can undergo large accelerations and decelerations. The response time for this kind of system is generally very good.

## Elastic systems

For systems where the coupling is elastic, mechanical resonance phenomena occur. Imagine that the rotor is free of any movement (no command is applied). The resonance frequency of the system is calculated as follow.

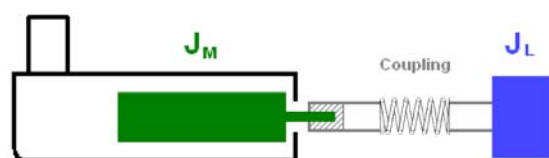


Figure 3: Inertias in movement with elastic coupling

# Inertia issue

$$f_r = \frac{1}{2\pi} \sqrt{k \left( \frac{1}{J_M} + \frac{1}{J_L} \right)}$$

Where:

- $f_r$  The mechanical resonance frequency in [Hz]  
 $J_L$  The load inertia in [kg.m<sup>2</sup>]  
 $J_M$  The motor inertia in [kg.m<sup>2</sup>]  
 $k$  The coefficient of elasticity of the coupling [N/m]

When the motor is controlled, the resonant frequency is changed depending on the structure and the tuning of control loops.

This kind of harmful frequency can be compensated by two ways. The first method is to place into the control loops some notch filters that reduce the oscillations. The second method is a natural phenomenon of compensation of the resonant frequency through the inertia ratio. Indeed the loopback of the control system generates a cancellation of a certain frequency. When the inertia ratio is low (close to 1:1), the cancelled frequency is near the resonant frequency.

It is therefore obvious that having a high inertia ratio in an elastic system is not good for the system response. Nevertheless, whatever is the method to cancel the resonant frequency, these problems from elasticity limit the response time of the system.

# Mechanical systems to the test

## Rigid system

The round table (figure 4) consists of a motor coupled to a disk with or without gearbox. The disc has an inertia of 50 [kg.cm<sup>2</sup>]. The control and measurements are performed by the motor encoder.

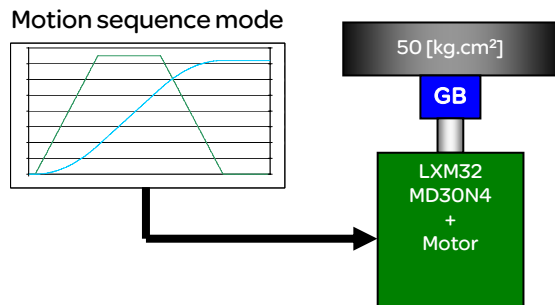


Figure 4: Round table Schema

A BSH motor and a BMH motor are compared with three different reduction ratios. The Lexium 32 controls the motor in order to follow a predefined trajectory in a "Motion Sequence".

## Elastic system

The linear belt axis (figure 5) consists of a motor coupled to the belt with or without gearbox. The belt moves a carriage bearing 6.5 [kg]. The inertia of the system in movement is estimated at 51.1 [kg.cm<sup>2</sup>]. The control and measurements are performed by the machine encoder to provide more precision in this type of elastic coupling.

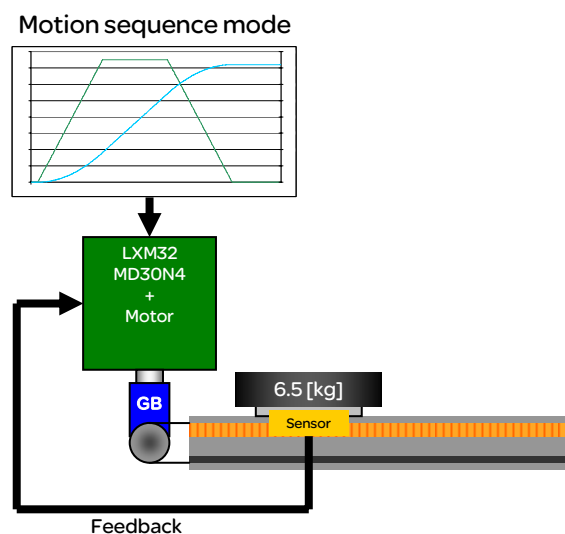


Figure 5: Belt axis schema

Like for the rigid system, a BSH motor and a BMH motor are compared with three different

reduction ratios. The Lexium 32 controls the motor in order to follow a predefined trajectory in a "Motion Sequence".

## Test sequence description

The trajectory is the same for tests on the two mechanical systems. The goal is to shift the carriage of 500 [mm], with a speed limited to 3.1 [m/s] and acceleration/deceleration ramps of 38.75 [m/s<sup>2</sup>]. These values have been calculated so that the motors operate in their linear range of operation.

For the turntable, this means a rotation of 1161 [°], with a speed limited to 1200 [RPM] and acceleration/deceleration ramps of 15000 [rpm/s].

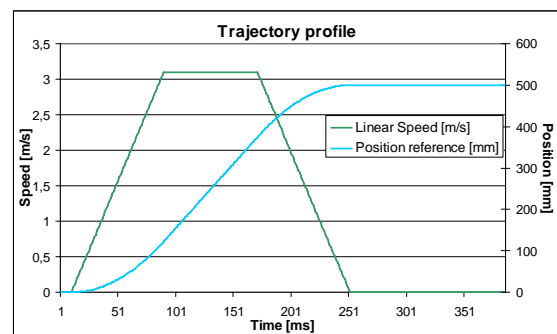


Figure 6: Belt axis sequence

The left scale represents the linear speed in [m/s], while the right scale represents the position of the carriage in [mm]. The objective is to reach the set-point without static error, with only one overshoot. The response time is here defined as the time which is necessary for the system to reach the set-point 1 0.1 [mm], from the time the reference has reached 500 [mm] in the error window.

For the turntable, the response time is defined as the time which is necessary for the system to reach the set-point 1 0.23 [°], from the time the reference has reached 1161 [°] in the error window.

## Motors

Both motors under tests are from two separated ranges, differentiated by the inertia. The BSH motors are called "low" inertia motors, whereas the BMH motors are called "medium" inertia motors.

# Mechanical systems to the test

Type of servo-motor	BSH0703P	BMH0703T
Switching frequency [kHz]	8	8
Continuous stall torque [Nm]	3.1	3.4
Peak stall torque [Nm]	11.3	10.2
Nominal torque [Nm]	2.44	2.9
Nominal speed [rpm]	5000	3000
Nominal servo motor output power [W]	1300	900
Maximum current [Arms]	17	17.8
Maximum mechanical speed [rpm]	8000	8000
Torque constant [Nm / Arms]	0.78	0.61
Back emf constant [Vrms / krpm]	49	39.3
Number of poles [.]	6	10
Inertia (without brake) [kgcm <sup>2</sup> ]	0.58	1.67
Resistance (phase / phase) [y]	2.7	1.32
Inductance (phase / phase) [mH]	13	4.3

Both product lines coexist in order to cover different applications. BSH motors, thanks to their low inertia are effective on high dynamic applications. As for BMH, they are dedicated to movement of heavier loads. Their larger inertia compensates phenomena of oscillations.

## Gearboxes

The speed and acceleration/deceleration ramps of the motor depend on the gearbox which is used, so that the conditions on the load are met. Three types of ratio are used:

- No gearbox, ratio of 1:1
- With a gearbox, ratio of 3:1
- With a gearbox, ratio of 5:1

The load inertia is divided by the square of the reduction ratio. The image of the disc inertia reported to the motor can be calculated, and then the different inertia ratios of the rigid system can be deduced.

Rigid system		
Reduction ratio	Theoretical inertia ratio with the BSH	Theoretical inertia ratio with the BMH
1:1	86:1	30:1
3:1	9.6:1	3.33:1
5:1	3.45:1	1.20:1

The same calculation is done for the elastic system.

Rigid system		
Reduction ratio	Theoretical inertia ratio with the BSH	Theoretical inertia ratio with the BMH
1:1	88:1	31:1
3:1	9.8:1	3.40:1
5:1	3.52:1	1.22:1

# Outcome performance

## Rigid System

Response times versus the measured inertia ratio are summarized in the chart below.

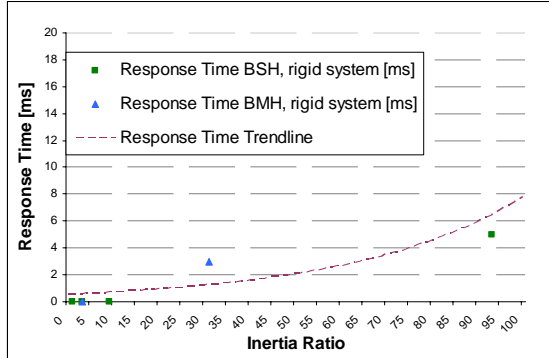


Figure 7: Response time versus the inertia ratio for the rigid system

For low inertia ratios, until 10:1, the response time is less than a millisecond because the response follows exactly the reference. When the inertia ratio increases, oscillations appear gradually. Even if the system is "rigid", materials are deformed and have one or more resonant frequencies. The response time increases, since the response is overshooting. In the worst case, the response time is 5 [ms] for a measured inertia ratio of 93:1. This is quite acceptable in most applications since this time represents only 2% of the rising time of 240 [ms].

## Elastic system

Response times versus the measured inertia ratio are summarized in the chart below.

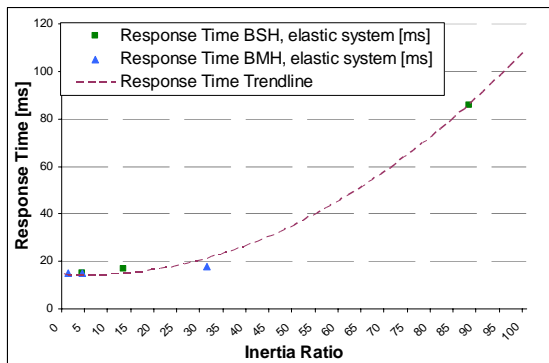


Figure 8: Response time versus the inertia ratio for the elastic system

The lowest inertia ratio is of 1.36:1; in this configuration the response time is 15 [ms]. It seems that the mechanics do not allow going below this value.

Gearboxes increase the motor inertia from the point of view of the load, making the command

more robust and the resonance phenomena better controlled.

Beyond an inertia ratio of 40:1, the response time of the system is rapidly deteriorated. Of course it depends on the rigidity of the belt. A more rigid belt allows better performance. The measurements indicate that for this test, the elastic coefficient "k" is approximately 658 700 [N/m] for a length of 2 [m].

The difference between the two motor ranges is very obvious since without gearbox, the BSH motor has a response time of 86 [ms] because of the oscillations. The BMH motor, thanks to its greater inertia, compensates these oscillations and managed to stabilize the position in 18 [ms].

## Comparisons

The purpose of this section is to determinate a limit criterion of inertia ratio not to exceed. This criterion depends on the type of controlled system.

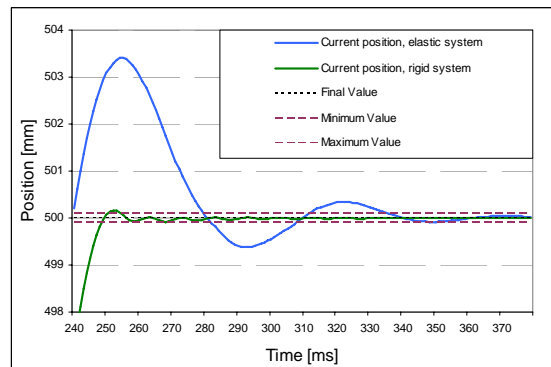


Figure 9: Comparison between the rigid and the elastic system for a high inertia ratio (BSH0703P motor without gearbox)

The oscillations of the elastic system have a low frequency with high amplitude compared to the rigid system. As we have seen in the theoretical calculation, more the system is rigid, more the resonant frequency is high.

To define the criterion, the maximum response time is set at 24 [ms], which represents 10% of the rising time. For a back and forth sequence, with a perfect system (response time of 0 [ms]), the working rate would be of 125 [strokes/min]. With a response time of 24 [ms], the working rate reduces to 113.64 [strokes/min], which represent a diminution of 9%. The chart of the response times for both systems allows deducing a limit.

# Outcome performance

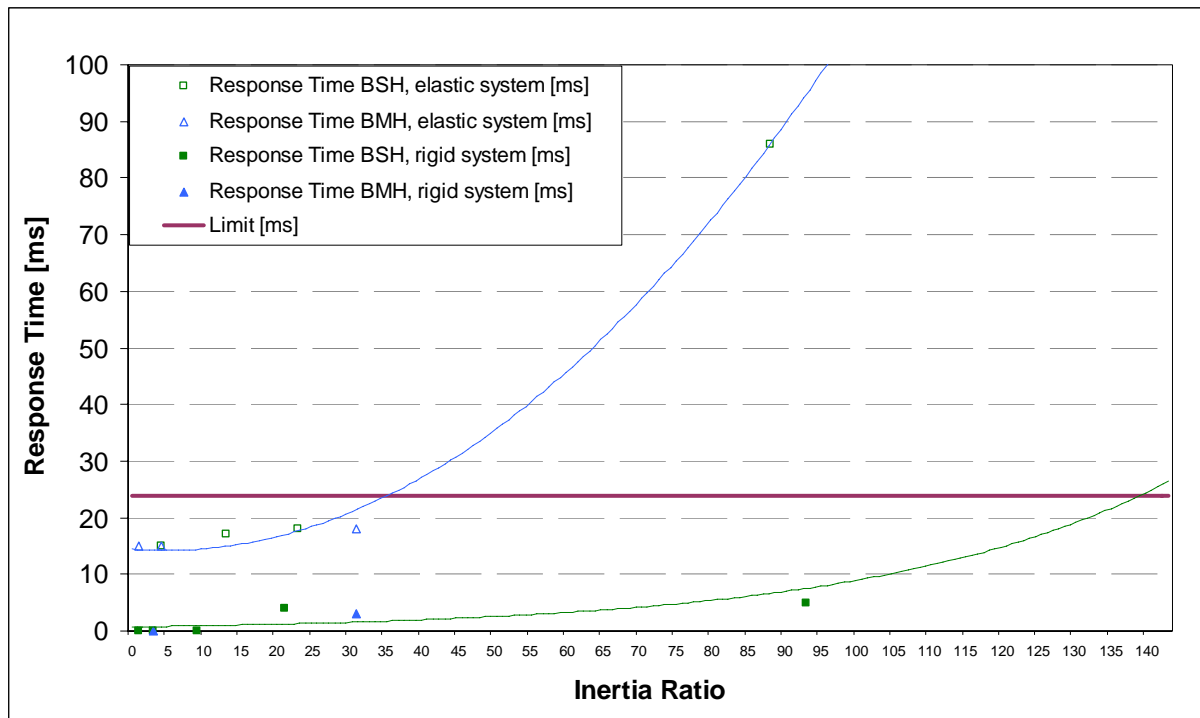


Figure 10: Response time versus inertia ratio

The two measures added with an inertia ratio around 22:1 are performed on a BSH1002P motor to obtain a better precision in the trend lines.

The limit inertia ratio is of 35:1 for the elastic system, and of 140:1 for the rigid system.



# Energy consumption

## Issue

The addition of a gearbox increases significantly the energy required to move the load. Indeed, if the load is always moved at the same speed, the motor should rotate faster with stronger acceleration/deceleration ramps. During the tests, the mass accelerates to reach a constant speed, and then decelerates to stop. The energy spent for the entire movement is as follows (for a system without gearbox).

$$\begin{aligned}
 E_{Theoretical} &= \frac{1}{2} J_T (\omega - 0)^2 + \frac{1}{2} J_T (0 - \omega)^2 \\
 &= J_T (\omega)^2 \\
 &= (J_M + J_L)(\omega)^2
 \end{aligned}$$

Where:

- $E_{Theoretical}$  The theoretical energy required for the movement in [J]
- $J_T$  The total system inertia in [kg.m<sup>2</sup>]
- $\omega$  The final rotation speed of the motor in [rad/s]
- $J_L$  The load inertia in [kg.m<sup>2</sup>]
- $J_M$  The motor inertia in [kg.m<sup>2</sup>]

With the addition of a gearbox, the motor speed must be increased, but the image of the load inertia is reduced.

$$\begin{aligned}
 E_{Theoretical} &= (J_M + \frac{J_L}{R^2})(\omega R)^2 \\
 &= J_M (\omega R)^2 + J_L (\omega)^2
 \end{aligned}$$

Where:

- $E_{Theoretical}$  The theoretical energy required for the movement in [J]
- $\omega$  The final rotation speed of the motor in [rad/s]
- $J_L$  The load inertia in [kg.m<sup>2</sup>]
- $J_M$  The motor inertia in [kg.m<sup>2</sup>]
- $R$  The reduction ratio [ ]

The energy is split into two terms, one is constant and the other depends on the gearbox. The first represents the energy required to move the load that is constant because the movement is identical with or without gearbox. The second represents the energy required to move the rotor, where the

speed increases proportionally with the reduction ratio.

## Results

The different measures of energy consumption according to the reduction ratio are summarized in the chart below.

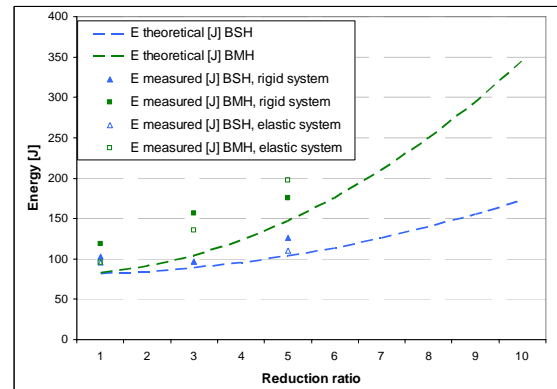


Figure 11: Energy consumption versus reduction ratio

The measured energy is greater than the theoretical energy since the system includes friction: even at constant speed the motor spends some energy. The growth of energy is not negligible since for the BMH motor with the elastic coupling, consumption is more than doubled between the configuration without gearbox and the configuration with the 5:1 gearbox.

BMH motors, because of their greater inertia, have a higher electrical consumption but their response time is shorter because the inertia ratio is more advantageous.

## Concrete example

To better understand these differences in terms of consumption and performance, a sequence is executed. The principle is to do a back and forth to link two defined positions, as quick as possible.

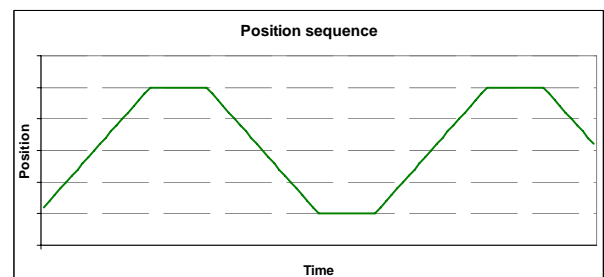


Figure 12: Repeated sequence

# Energy consumption

The rising time is determined by the sequence of previous tests, the stabilization time depends on the gearbox. The number of back and forth per minute and the electrical consumption are deduced. The studied system is the linear belt axis of 2 [m] with the BMH motor. The position to reach is still 500 [mm]. Once the position is reached and stable, a 10 [ms] delay is added to simulate an action.

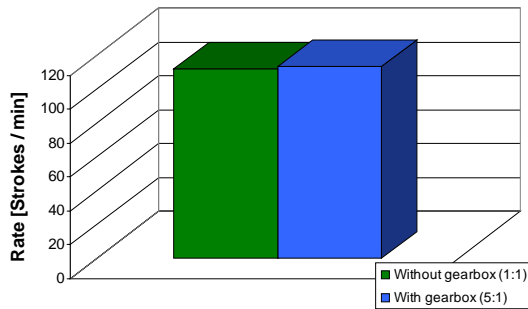


Figure 13: Work rate versus reduction ratio

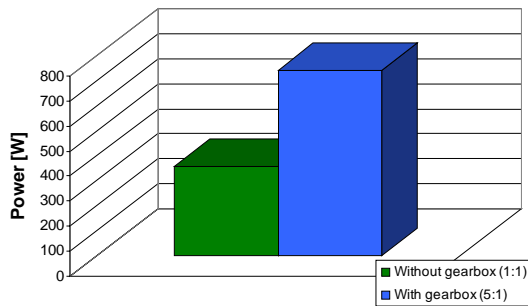


Figure 14: Electrical consumption versus reduction ratio

The addition of the gearbox allows gaining a few more than one stroke per minute, which represents only 1.1% of gain. However, the difference concerning consumption is much more obvious since the average power increased from 358 [W] to 743 [W] with the addition of the gearbox. This represents a consumption raise of 108%.

# Conclusion

This is not an easy thing to conclude on the inertia ratio to recommend in the general case. This heavily depends on the elasticity of the system and performance which are expected. In theory, for rigid systems the response depends on the maximum torque developed by the motor. In practice, any coupling is more or less elastic and therefore includes phenomena of resonance, which deteriorate the response time.

The limit criterion for the inertia ratio, which is deduced from the tests, is of 35:1 for the linear belt axis, and 140:1 for the round table.

The issue of gearbox is more obvious: by maintaining an inertia ratio below a limit allowing to reach the expected performance, the reduction ratio must be chosen as low as possible to minimize the energy consumed. Since this energy increases with the square of the reduction ratio, it is possible to save a lot by carefully choosing its gearbox.